J. Appl. Maths Mechs Vol. 57, No. 2, pp. 355–362, 1993
Printed in Great Britain.

0021–8928/93 $24.00+.00

A PIECEWISE-HOMOGENEOUS ELASTIC PLANE WITH A COUNTABLE SET OF CLOSED CRACKS†

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(*Received* 18 *February* 1991)

The stress–strain state of a piecewise-homogeneous elastic plane composed of two distinct half-planes whose common boundary contains a denumerable set of closed cracks which become more dense towards infinity on one or both sides is considered. The shear stresses at the crack edges and the stress and rotation at infinity are specified.

Using the Riemann boundary-value problem method for a denumerable set of contours, complex potentials for the problem are constructed and formulae are obtained for the stress intensity factor. The problem of the interaction of a closed macro-crack with an infinite row of closed micro-cracks collinear with it is also studied.

SPECIAL cases of the problem under consideration have been studied previously in [1–4], namely, homogeneous or piecewise-homogeneous planes with finite or infinite periodic systems of closed cracks. In the latter case, the stress at the crack edges was, in general, non-periodic.

1. STATEMENT OF THE PROBLEM

Suppose that the elastic $z=x+iy$ plane is bonded together from homogeneous isotropic upper and lower half-planes with different elastic characteristics, with a denumerable number of closed cracks $L_n=[a_n,\ b_n]$, $n\in I$ situated along the interface of the media, becoming more dense towards infinity and satisfying the conditions

$$b_n-a_n\leqslant D<\infty,\quad a_{n+1}-a_n\geqslant d>0,\quad n\in I \tag{1.1}$$

where the set of indices $I=\{0,\ \pm 1,\ \ldots\}$ or $I=\{0,\ 1,\ \ldots\}$. In the first case the cracks become more dense towards infinity on both sides, and in the second case, to the point $+\infty$. Conditions (1.1) are satisfied, for example, by cracks situated periodically along the entire real axis or just a semi-axis.

Suppose

1. the sides $L_n^\pm$ of the cracks are acted upon by given shear stresses $\tau_{xy}^\pm(t)=h^\pm(t)$, $t\in L_n$, Hölder-continuous in each interval L_n and decreasing as $O(t^{-\lambda})$ when $t\to\infty$, $\lambda>1$;
2. the normal stress σ_y and the normal displacement υ in the transition from one edge of the crack to the other change continuously, i.e. $\sigma_y^+(t)=\sigma_y^-(t)$, $\upsilon^+(t)=\upsilon^-(t)$, $t\in L_n$; and
3. outside the cracks along the interface of the media there is total adhesion.

In the case under consideration, the stresses σ_x, σ_y, τ_{xy}, the rotation ω and the partial

†*Prikl. Mat. Mekh.* Vol. 57, No. 2, pp. 133–140, 1993.

derivatives u', υ' with respect to x of the displacement components are expressed in terms of two piecewise-holomorphic functions $\Phi_{1,2}(z)$, which have lines of discontinuity $L=\bigcup_{n\in I} L_n$ and are called complex potentials, by the formulae [5]

$$\begin{aligned}
&\tfrac{1}{4}(\sigma_x+\sigma_y)+2i\mu_j(\kappa_j+1)^{-1}\omega=r_j\Phi_1(z)+r_{j+2}\Phi_2(z)\\
&\sigma_y-i\tau_{xy}=r_j\Phi_1(z)+r_{j+2}\Phi_2(z)+\Omega_j(z)\\
&2\mu_j(u'+i\upsilon')=\kappa_j[r_j\Phi_1(z)+r_{j+2}\Phi_2(z)]-\Omega_j(z),\quad j=1,2\\
&\Omega_j(z)=r_j(z-\bar z)\overline{\Phi_1'(z)}+r_{3-j}(\bar z)+r_{j+2}[(z-\bar z)\overline{\Phi_2'(z)}-\Phi_2(\bar z)]\\
&r_1=(\kappa_1+m)^{-1},\quad r_2=(1+m\kappa_2)^{-1},\quad r_3=m(1+\kappa_2),\quad r_4=1+\kappa_1,\quad m=\mu_1\mu_2^{-1}
\end{aligned}\tag{1.2}$$

where the index $j=1$ refers to the upper half-plane and the index $j=2$ to the lower half-plane. Here μ_j is the shear modulus, $\kappa_j=3-4\nu_j$ in the case of plane deformation and $\kappa_j=(3-\nu_j)/(1+\nu_j)$ in the case of the plane stress state, and ν_j is Poisson's ratio.

Problem. Find the stress–strain state of a plane with cracks L_n, $n\in I$, given by formulae (1.2), when the complex potentials $\Phi_{1,2}(z)$ can become infinite with order less than one at the tips of the cracks, which at points t of the line L other than at the ends satisfy the condition

$$\lim(z-\bar z)\Phi_{1,2}'(z)=0\quad\text{as}\quad z\to t^{\pm}\tag{1.3}$$

and for which large z situated outside some fixed small neighbourhood $U(L)$ of the line L satisfy the inequalities

$$|\Phi_{1,2}(z)|\leqslant M|z|^{-\lambda},\quad |(z-\bar z)\Phi_{1,2}'(z)|\leqslant M|z|^{-\lambda},\quad M=\text{const},\quad \lambda>1\tag{1.4}$$

Obviously the same inequalities will also be satisfied by the stresses at large $z\notin U(L)$. In this case, as in the case of fundamental problems of the theory of elasticity for a plane with a finite number of cuts [6], one can show that the solution of the problem under consideration will be unique, if it exists.

From the boundary conditions on the line L

$$\tau_{xy}^{\pm}(t)=h^{\pm}(t),\quad \sigma_y^{+}(t)=\sigma_y^{-}(t),\quad \upsilon^{+}(t)=\upsilon^{-}(t)$$

and on the basis of Eqs (1.2) and (1.3) we obtain the boundary-value problems

$$\operatorname{Im}\Phi_1^{+}(t)=g(t)+r_2h(t),\quad \operatorname{Im}\Phi_1^{-}(t)=g(t)-r_1h(t),\quad t\in L\tag{1.5}$$

$$\Phi_2^{+}(t)-\Phi_2^{-}(t)=if(t),\quad t\in L\tag{1.6}$$

$$\begin{aligned}
&f(t)=(h^{-}(t)-h^{+}(t))/(r_3+r_4),\quad g(t)=-(r_4h^{+}(t)+r_3h^{-}(t))/(r_3+r_4)\\
&h(t)=m(1-\kappa_1\kappa_2)f(t)/(r_1+r_2)
\end{aligned}$$

Writing down the equilibrium condition for the part of the cracked plane bounded by the circle $|z|\leqslant R$, and taking the limit as $R\to\infty$, we find that in the case under consideration the principal vector P for the tangential forces acting on the sides of all the cuts is zero, i.e.

$$\int_L f(t)dt=-(r_3+r_4)^{-1}P=0\tag{1.7}$$

We will assume that this necessary condition for the problem to be solvable is satisfied.

2. SOLUTION OF THE PROBLEM

The solution of problem (1.6) has the form [7]

$$\Phi_2(z)=\frac{1}{2\pi}\int_L\frac{f(t)dt}{t-z}=\sum_{n\in I}\frac{1}{2\pi}\int_{L_n}\frac{f(t)dt}{t-z} \tag{2.1}$$

where the improper integral over L and the series converge absolutely and uniformly in z in any closed bounded domain not containing points of the line L. From (1.7) we have

$$\Phi_2(z)=\frac{1}{2\pi z}\int_L\frac{tf(t)dt}{t-z}-\frac{1}{2\pi z}\int_L f(t)dt=\frac{1}{2\pi z}\int_L\frac{tf(t)dt}{t-z} \tag{2.2}$$

Since the function $tf(t)$ decreases as $O(t^{1-\lambda})$, $\lambda>1$ as $t\to\infty$, for large $z\notin U(L)$ the function $\Phi_2(z)$ satisfies inequalities (1.4)

The solution of problem (1.5) has the form [8] $\Phi_1(z)=\Psi_1(z)+\Psi_2(z)$ where the function $\Phi_1(z)$ satisfies the conditions

$$\Psi_1^+(t)+\Psi_1^-(t)=ip(t),\quad p(t)=2g(t)+(r_2-r_1)h(t),\quad t\in L \tag{2.3}$$

$$\overline{\Psi_1(\bar z)}=-\Psi_1(z),\quad z\notin L \tag{2.4}$$

and $\Psi_2(z)$ is the solution of the problem

$$\Psi_2^+(t)-\Psi_2^-(t)=im(1-\kappa_1\kappa_2)f(t),\quad t\in L$$

$$\overline{\Psi_2(\bar z)}=\Psi_1(z),\quad z\notin L$$

whence $\Psi_2(z)=m(1-\kappa_1\kappa_2)\Psi_2(z)$. As a consequence

$$\Phi_1(z)=\Psi_1(z)+m(1-\kappa_1\kappa_2)\Phi_2(z) \tag{2.5}$$

We take a particular solution of problem (2.3) of the form

$$\Psi_0(z)=X(z)F(z)$$

$$X(z)=\prod_{n\in I}\frac{z-c_n}{[(z-a_n)(z-b_n)]^{1/2}},\quad c_n=\frac{a_n+b_n}{2} \tag{2.6}$$

$$F(z)=\sum_{n\in I}F_n(z),\quad F_n(z)=\frac{1}{2\pi(z-c_n)}\int_{L_n}\frac{t-c_n}{X^+(t)}\frac{p(t)dt}{t-z} \tag{2.7}$$

where by virtue of (1.1) the infinite product and the series converge absolutely and uniformly in any closed bounded domain not containing points of the line L, and $X(z)$ is taken to be the branch that is single-valued in the plane with cuts along the line L and has a limit equal to unity as $z=iy\to\pm i\infty$. At the points c_n, $n\in I$ the function $X(z)$ has first-order zeros. Outside the neighbourhood $U(L)$ it satisfies the inequalities

$$0<M_1\leq|X(z)|\leq M_2<\infty,\quad |(z-\bar z)X'(z)|\leq M_2$$

and the function $F(z)$ satisfies inequalities (1.4) Then the function $Q(z)=(\Psi_0(z)-\Psi_1(z))/(iX(z))$ is meromorphic with simple poles c_n, $n\in I$ and for large $z\notin U(L)$ satisfies inequalities (1.4), and therefore [9]

$$Q(z)=\sum_{n\in I}A_n/(z-c_n) \tag{2.8}$$

where the numbers A_n are such that the series converges uniformly in any closed bounded domain not containing any points c_n. Consequently

$$\Psi_1(z)=X(z)[F(z)-iQ(z)] \tag{2.9}$$

Since $\overline{X(\overline{z})}=X(z)$, and the function $F(z)$ like the function $\Psi_1(z)$, satisfies condition (2.4), we have $\overline{Q(\overline{z})}=Q(z)$, for which it is necessary and sufficient that the numbers A_n are real. From Eqs (1.2), (1.6), (2.3) and (2.5) the single-valuedness of the displacements during a passage around the cracks is expressed by the conditions

$$\operatorname{Re}\int_{L_n}\psi_1^+(t)dt=0,\quad n\in I$$

whence, substituting for $\Psi_1^+(t)$, we obtain an infinite system of linear algebraic equations for finding the constants A_n

$$\begin{gathered}\sum_{k\in I}\delta_{nk}A_k=H_n,\quad n\in I\\ \delta_{nk}=i\int_{L_n}X^+(t)(t-c_k)^{-1}dt,\quad H_n=\int_{L_n}X^+(t)F(t)dt\end{gathered} \tag{2.10}$$

where $F(t)$ is taken to be the principal value of integral (1.7) Because the functions $X^+(t)$, $F(t)$ take purely imaginary values on L_n, the numbers $\delta_{n,k}$ and H_n are real, and for large n the H_n satisfy the inequality $|H_n|\leq M|n|^{-\lambda}$, $\lambda>1$.

A solution of system (2.10) should be sought in the class Π of real sequences A_n, $n\in I$ such that series (2.8) converges uniformly and defines the function $Q(z)$ in any closed bounded domain not containing any points c_n, the function $Q(z)$ satisfying inequalities (1.4) for large $z\notin U(L)$. From the uniqueness of the solution of the elasticity problem it follows that if system (2.10) is solvable in the specified class of sequences, then its solution is unique. The solvability of this system for certain crack distributions has been proved by methods of functional analysis, where the solution of the system is found by the method of successive approximations or the reduction method, and for large n it satisfies the inequality $|A_n|\leq M|n|^{-\lambda}$, $\lambda>1$. This occurs, for example, in the following cases [10, 11]:

A. $L_n=[n\pi-a,\ n\pi+a]$, $n=0,\pm1,\ldots$, i.e. the cracks are arranged periodically along the entire real axis. In this case

$$X(z)=(\sin^2 z-\sin^2 a)^{-1/2}\sin z \tag{2.11}$$

and system (2.10) has the form

$$\sum_{k=-\infty}^{\infty}\delta_{n-k}A_k=H_n,\quad n=0,\pm1,\ldots$$

$$\delta_n=\int_0^a\frac{2x}{x^2-n^2\pi^2}\frac{\sin x\ dx}{(\sin^2 a-\sin^2 x)^{1/2}}$$

from which [12, 13]

$$A_n=\frac{1}{2\pi i}\int_{|t|=1}\frac{H(t)}{\delta(t)}\frac{dt}{t^{n+1}}=\sum_{k=-\infty}^{\infty}\xi_{n-k}H_k$$

$$H(t)=\sum_{n=-\infty}^{\infty}H_nt^n,\quad \delta(t)=\sum_{n=-\infty}^{\infty}\delta_nt^n$$

where the ξ_n are the Fourier coefficients of the function $1/\delta(t)$.

B. $L_n=[n-a,\ n+a]$, $n=0, 1, \dots$, i.e. the cracks are arranged periodically only along the real semi-axis. In this case

$$X(z)=(\Gamma(a-z)\Gamma(-a-z))^{1/2}/\Gamma(-z) \tag{2.12}$$

C. Some of the cracks are situated periodically with a single period along the negative real semi-axis, others are situated periodically with a different period along the positive semi-axis, and the remainder, comprising no more than a finite number of cracks, are non-periodic. In this case the function $X(z)$ is also expressible in terms of gamma functions.

D. The cracks are such that the inequalities $a_{n+1}\geqslant da_n$, $a_{-n-1}\leqslant da_{-n}$, $d=\mathrm{const}>1$ are satisfied for sufficiently large positive n.

The question as to whether system (2.10) is solvable in the sequence class Π for a general distribution of cracks remains open.

From formulae (1.2), (2.1) and (2.5)–(2.9) it is clear that the stress intensity near the ends $g_n=a_n$ or $g_n=b_n$ of the cracks L_n is given, to within $\ln|z-g_n|$ by the function $\Phi_1(z)$ which has the form [8]

$$\Phi_1(z)=-iK_n^{\pm}(r_1+r_2)^{-1}[\pm 2\pi(z-g_n)]^{-1/2}+O(1) \tag{2.13}$$

$$K_n^{\pm}=(r_1+r_2)\sqrt{\pi}\eta_n^{\pm}[iF(g_n)+Q(g_n)] \tag{2.14}$$

$$\eta_n^{\pm}=\lim_{z\to g_n}[\pm 2(z-g_n)]^{1/2}X(z)$$

where the upper sign refers to the right edge $c_n=a_n$ and the lower sign refers to the left one $g_n=a_n$. The real number $K_n^{\pm}$ in (2.13) is the stress intensity factor (SIF) in the form of [4]. In particular, according to (2.11) and (2.12), in cases A and B

$$K_n^{\pm}=(r_1+r_2)(\pi\,\mathrm{tg}\,a)^{1/2}[iF(n\pi\pm a)+Q(n\pi\pm a)] \tag{2.15}$$

and

$$K_n^{\pm}=(r_1+r_2)\Gamma(n+1\pm a)\left(\frac{\mathrm{tg}\,\pi a}{n!\Gamma(n+1\pm 2a)}\right)^{1/2}[iF(n\pm a)+Q(n\pm a)] \tag{2.16}$$

respectively.

Because the numbers $F(g_n)$ and $Q(g_n)$ decrease as $O(n^{-\lambda})$, $\lambda>1$ as $n\to\infty$, and the numbers $\eta_n^{\pm}$ are bounded by the same constant, the SIFs $K_n^{\pm}$ also decrease as $O(n^{-\lambda})$, $\lambda>1$ as $n\to\infty$.

The investigations of this section are summarized in the following theorem.

Theorem 1. The solvability of the elasticity problem posed in Sec. 1 is equivalent to the solvability of system (2.10) in the class of real sequences Π. In cases when the problem is solvable (for example, cases A–D), its solution is unique and is given by functions $\Phi_{1,2}(z)$ which are found from formulae (2.1), (2.5)–(2.9), and the SIF near the ends $g_n=a_n$ and $g_n=b_n$ of the cracks L_n are found from formulae (2.14)–(2.16) and when $n\to\infty$ they decrease as $O(n^{-\lambda})$, $\lambda>1$.

3. EXTENDING THE CLASS OF SOLUTIONS

Suppose that the principal vector P of the shear loads acting on the sides of the cracks is allowed to be non-zero. Then according to the first equality of (1.7) and equality (2.2), for large

$z \notin U(L)$ the function $\Phi_2(z) = Az^{-1} + O(z^{-\lambda})$, while the function $\Psi_1(z) = O(z^{-\lambda})$, where $A = P/(2\pi(r_3 + r_4))$ and $\lambda > 1$ From the above and the results of Sec. 2 we consider the stress–strain state, given by formulae (1.2) in terms of the functions $\Phi_{1,2}(z)$, which for large $z \notin U(L)$ have the form

$$\Phi_1(z) = \alpha - i\beta X(z) + m(1 - \kappa_1\kappa_2)Az^{-1} + O(z^{-\lambda}) \tag{3.1}$$
$$\Phi_2(z) = B + Az^{-1} + O(z^{-\lambda}), \quad \lambda > 1$$

where α and β are real and B is a complex constant. From formulae (1.2) we find

$$\alpha = \sigma_y^\infty / (r_1 + r_2), \quad \beta = \tau_{xy}^\infty / (r_1 + r_2) \tag{3.2}$$
$$B = [\tfrac{1}{4}(\sigma_x^\infty + \sigma_y^\infty) + 2i\mu_1(\kappa_1 + 1)^{-1}\omega^\infty - r_1(r_1 + r_2)^{-1}(\sigma_y^\infty - i\tau_{xy}^\infty)] / r_3$$

where σ_x^∞, σ_y^∞, τ_{xy}^∞, and ω^∞ are the stresses and rotation as $z = iy \to +i\infty$ whose values are to be specified. They are specified independently of the way the cracks are distributed. In this case the solution of the elasticity problem, assuming it is solvable, is also unique and is given by the functions

$$\Phi_1(z) = \alpha + X(z)[F(z) - iQ(z) - i\beta] + m(1 - \kappa_1\kappa_2)F_0(z) \tag{3.3}$$
$$\Phi_2(z) = B + F_0(z), \quad F_0(z) = \frac{1}{2\pi}\int_L \frac{f(t)dt}{t - z}$$

The functions X, F and Q are found from formulae (2.6)–(2.10), where in (2.10) it is necessary to put

$$H_n = \int_{L_n} X^+(t)(F(t) - i\beta)dt \tag{3.4}$$

Here in formulae (2.14)–(2.16) for the SIF it is necessary to add a term β within the square brackets. Since the numbers $n_n^\pm$ are bounded by the same constant, the SIF $K_n^\pm$ are also bounded by the same constant. We note that they do not depend on the values of σ_x^∞, σ_y^∞, and ω^∞

We will summarize the investigations of this section in the following theorem.

Theorem 2. The stress–strain state of a plane with closed cracks L_n, $n \in I$ is given by formulae (1.2) in terms of functions $\Phi_{1,2}(z)$ which for large $z \notin U(L)$ have the form (3.1). This state exists if and only if system (2.10) with right-hand sides (3.4) is solvable in the class of real sequences Π. If the problem is solvable (for example, cases A–D of Sec. 2), its solution is unique and is given by the functions (3.3), while the SIF are found from formulae (2.14)–(2.16), where it is necessary to add the term $\tau_{xy}^\infty/(r_1 + r_2)$ within the square brackets, and they have the same bounds.

In particular, if at infinity as $u \to \infty$ there are stresses σ_x^∞, σ_y^∞, and τ_{xy}^∞, rotation ω^∞ and $h^\pm(t) \equiv 0$, i.e. the crack edges slide past each other without friction, then the solution of the problem, when it is solvable, is given by the functions

$$\Phi_1(z) = \alpha - i\beta X(z)\left[1 - \sum_{n \in I} A_n / (z - c_n)\right], \quad \Phi_2(z) = B$$

where the function X and constants α, β and B are found from formulae (2.6) and (3.2), respectively, while A_n, $n \in I$ is the solution of system (2.10) in the special case when

$$H_n = i\int_{L_n} X^+(t)dt \tag{3.5}$$

In this case the SIF near the tips of the crack $g_n = a_n$ and $g_n = b_n$ is

$$K_n^{\pm} = \gamma(r_1 + r_2)^{-1}\tau_{xy}^{\infty}\left[1 - \sum_{k\in I} A_k(g_n - c_k)^{-1}\right]$$

where γ is the coefficient in front of the square brackets in (2.14)

In cases A and B described in Sec. 2, the SIF have the asymptotic representation

$$K_n^{\pm} = (\xi\,\mathrm{tg}(\pi a/\xi))^{1/2}\tau_{xy}^{\infty} + O(n^{-\lambda}),\quad \lambda > 1$$

as $n \to \infty$, with $\xi = \pi$ in case A and $\xi = 1$ in case B.

4. INTERACTION OF A MACRO-CRACK WITH AN INFINITE SERIES OF MICRO-CRACKS

Suppose that in addition to the closed cracks $L_n = [a_n, \; b_n]$, $(n = 0, 1, \ldots)$ satisfying conditions (1.1), the z plane also contains a single closed semi-infinite crack $L_{-1} = (-\infty, b]$, $b < a_0$. Along the sides of the latter we specify Hölder-continuous loads $\tau_{xy}^{\pm} = h^{\pm}(\tau)$ which decrease as $O(t^{-\lambda})$, $\lambda > 1$ as $t \to \infty$. In this case, under the restrictions imposed on the required solution in Sec. 1, all the results of Secs 1 and 2 hold, apart from formulae (2.6) and (2.7) which should be replaced with

$$\mathrm{X}(z) = (z - b)^{-1/2}\prod_{n=0}^{\infty}\frac{z - c_n}{[(z - a_n)(z - b_n)]^{1/2}} \tag{4.1}$$

$$F(z) = \frac{1}{2\pi}\int_{-\infty}^{b}\frac{p(t)}{\mathrm{X}^+(t)}\frac{dt}{t - z} + \sum_{n=0}^{\infty}F_n(z) \tag{4.2}$$

In all the remaining formulae one should take $L = L_{-1} \cup L_0 \cup L_1 \cup \ldots$, and the solution of system (2.10) should be sought in the class of real sequences A_n, $n \in I$, $I = \{0, 1, \ldots\}$ such that in any closed bounded domain not containing any points c_n the series (2.8) converges uniformly and defines the functions $Q(z)$ and $(z - \bar{z})Q'(z)$ which decrease as $O(z^{-\nu})$, $\nu > 1/2$ as $z \to \infty$ outside $U(L)$.

As in Sec. 3, we extend the class of solutions of the problem, requiring that the functions $\Phi_{1,2}(z)$ have the form (3.1) for $z \notin U(L)$. If it exists, the unique solution of the problem is given by formulae (3.3) where one must take $L = L_{-1} \cup L_0 \cup L_1 \cup \ldots$. For large z in the upper half-plane along the ray $\arg z = \theta$, we have according to (1.2), (3.1) and (4.1)

$$\tfrac{1}{4}(\sigma_x + \sigma_y) + 2i\mu_1(\kappa_1 + 1)^{-1}\omega = r_1\alpha + r_3B - i\beta r_1\rho^{-1/2}e^{-i\theta/2} + O(\rho^{-1})$$

$$\sigma_y - i\tau_{xy} = \alpha(r_1 + r_2) - i\beta\rho^{-1/2}(r_1e^{-i\theta/2} + r_2e^{i\theta/2}) + O(\rho^{-1}),\quad \rho = |z|$$

from which it follows that the stress τ_{xy} vanishes at infinity and the constants α and B are found from formulae (3.2) where $\tau_{xy}^{\infty} = 0$, and to find the real constant β it is necessary to specify to within $\rho^{-1/2}$ inclusive the behaviour of one of the stresses τ_{xy}, σ_y and $\sigma_x + \sigma_y$, as $z \to \infty$ along any ray.

Suppose, for example, that along the imaginary axis $\arg z = \pi/2$ for large $p = |z|$ the stress $\tau_{xy} = \tau_{xy}^0\rho^{-1/2} + O(\rho^{-1})$, where τ_{xy}^0 is a specified real number. Then $\beta = \sqrt{(2)}\tau_{xy}^0/(r_1 + r_2)$. In this case the problem has a solution that is non-vanishing at infinity, even if the stresses at the sides of all the cracks are zero, i.e. the problem is of class N [1]. This solution, which depends on the single real parameter β describing the stress intensity in a neighbourhood of infinity, is given by the functions

$$\Phi_1(z) = -i\beta X(z)[1 - Q(z)], \quad \Phi_2(z) = 0$$

where X and Q are found from formulae (4.1) and (2.8), (2.10) in which H_n should be taken in the form (3.5).

Remarks. 1. The results obtained above are still true in the case when the set I is finite, i.e. the number of cracks is finite. In this case the products (2.6), (4.1), the series (2.7), (2.8), (4.2) and system (2.10) are finite.

2. The results do not change if it is required that inequalities (1.4) are satisfied for large z not in the entire exterior of the neighbourhood $U(L)$ of the line L, but only on some system $C = \{C_j\}$, $j = 1, 2, \ldots$ of nested smooth curves such that the distance from the tips of the cracks to the points of system C is not less than some positive constant and that for all j the ratio of the length of the curve C_j to the shortest distance from its points to the origin is bounded by the same constant.

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Translated by R.L.Z.